Physics Factsheet

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Interpreting and Drawing Graphs of Motion

In Factsheet 116, we looked at graphical work with electricity. This was in response to Examiner's Reports suggesting that students have displayed weaknesses in sketching and interpreting graphs.

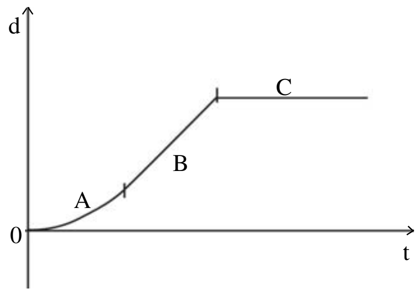
In this Factsheet we will follow this up with similar work within Motion – a topic that lends itself to graphical work. We will be looking at selected examples within this topic.

Velocity-time and displacement-time graphs

We usually use the terms **velocity** and **displacement** here because they are vector quantities (unlike **speed** and **distance**), and so can have a direction rather than just a magnitude.

Generally we look at linear motion, and positive and negative values represent the forward and backward directions.

Displacement-time (d-t)



This graph is divided into three sections.

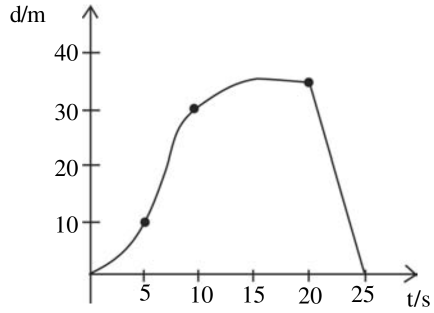
- In **A**, the rate of change of displacement with time is increasing. This implies acceleration.
- In **B**, the displacement increases at a steady rate. This implies uniform speed (or uniform velocity, more correctly).
- In **C**, there is no change in displacement. The object is stationary.

Example: In A, do we have uniform acceleration?

Answer: *There is not enough information to tell. But the fact that there is a smooth transition to uniform speed suggests that the acceleration is slowly decreasing to zero at the end of section **A**.*

Exam Hint: *Don't attempt to draw too many conclusions from sketch graphs. Without numerical data being provided, there is often not enough information to be certain about anything except trends.*

Considerably more detail is available in this graph:



(a) What is the average velocity for the journey?
Zero, as you end up back at the starting point (velocity is a vector)

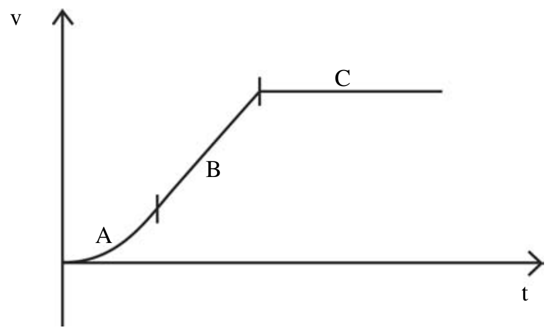
(b) What is the average speed for the journey?
$V_{ave} = 70/25 = 2.8 ms^{-1}$

(c) What is the average speed between the 5 and 10 second marks?
$V_{ave} = 20/5 = 4.0 ms^{-1}$

(d) What can we learn from the area under the graph?
Nothing of real value (displacement × time?)

(e) What would a negative value for displacement tell you?
That you were now behind your original starting position.

Velocity-time (v-t)

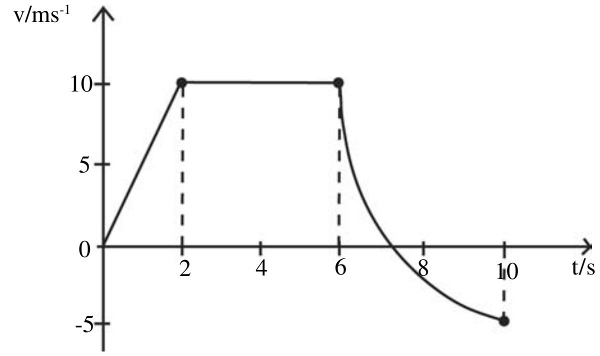


On first glance, this graph looks like the earlier one. But this is a velocity-time graph, not a displacement-time graph.

- In **A,** the rate of change of velocity with time is increasing. This shows **increasing** acceleration.
- In **B,** the velocity increases at a steady rate. This acceleration is **uniform**.
- In **C**, there is uniform velocity (as opposed to the object being stationary in the d-t graph).

Exam Hint: *An identical looking graph describes totally different motion. Do not confuse velocity-time and displacement-time graphs.*

Example: This v-t graph represents the motion of an object:

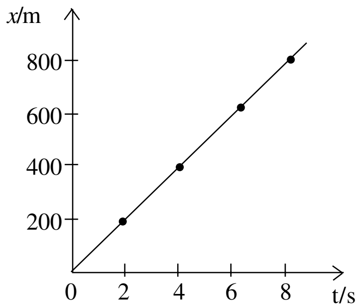


(a) Find the acceleration in the first 2 seconds.
(b) Find the distance travelled in the first 6 seconds.
(c) Estimate the **distance** travelled over the 10 seconds.
(d) Estimate the **displacement** over the 10 seconds.

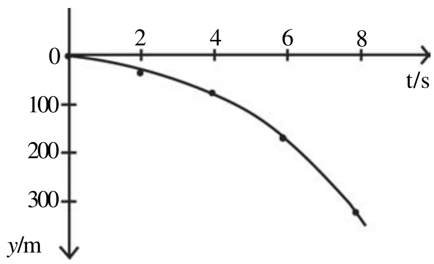
Answer
(a) $a = 10/2 = 5\text{ms}^{-2}$
(b) Distance = area under graph = 10 + 40 = 50m
(c) Estimate areas between 6 and 10 seconds.
Distance = 10 + 40 + 5 + 7 = 62m
(d) Subtract motion back towards start (negative velocity)
Displacement = 10 + 40 + 5 - 7 = 48m forwards.

Projectile motion graphs
A projectile, e.g. cannonball, travels in both the **x** and **y** planes. Its position also changes with time. We cannot put all three variables on one graph. But obtaining two graphs using our knowledge of physics, allows us to predict the third graph.

Let's look at a cannonball fired horizontally at 100ms^{-1} from a cliff.
(a) **x-direction**
Ignoring air resistance, it should travel at constant speed.



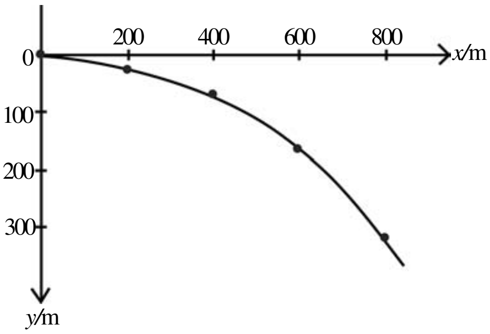
(b) **y-direction**
The Earth's gravitational field will cause an acceleration of approximately 10ms^{-2} vertically downwards.
After 2 seconds, $y = 0.5 \times 10 \times 2^2 = 20\text{m}$
After 4 seconds, $y = 0.5 \times 10 \times 4^2 = 80\text{m}$
After 6 seconds, y = 180m
After 8 seconds, y = 320m



We can now make a table of **x** and **y** at equal times:

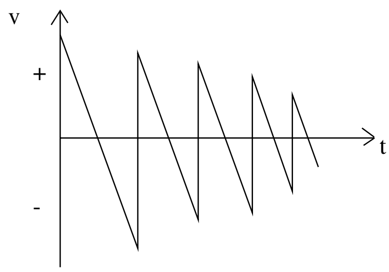
t/s	x/m	y/m
0	0	0
2	200	20
4	400	80
6	600	180
8	800	320

And then draw a graph of the cannonball's path:



It is often possible to use two relationships to determine a third. We generally see this in algebra, but it can be performed graphically as well.

We often see velocity-time graphs of a ball bouncing on a hard surface:



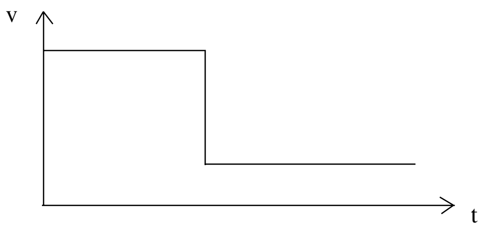
This graph displays a number of Physics ideas:
(a) Why is the downwards gradient always the same?
The rate of change of velocity is the acceleration due to the earth's gravitational field. This is a constant, of course (9.81ms^{-2}).
(b) Why is the upwards gradient almost vertical each time?
The sudden change of velocity occurs as the ball distorts, then bounces. This happens very quickly.
(c) Why does the amplitude decrease from bounce to bounce?
Kinetic energy is lost to heat energy as the ball distorts and bounces.
(d) Why is each positive peak followed by a negative peak of exactly the same displacement?
Ignoring air resistance, there should be no loss of energy as the ball rises, then falls, between bounces.
(e) How could you find the height reached each time?
For a v-t graph the distance travelled up or down would be the area under each positive or negative triangle.

Exam Hint: *Presented with graphs of this type, be prepared to perform calculations concerning acceleration, maximum height, kinetic energy (and gravitational potential energy), percentage of energy lost in each bounce, etc. (See the questions at the end of the Factsheet.)*

Collisions

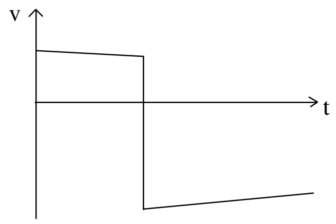
Momentum is conserved in collisions, but some kinetic energy is usually lost to heat and sound. As a result, we usually concentrate on conservation of momentum with collisions.

Suppose a moving trolley **A** collides with a stationary trolley **B**, and the pair move off together. Here is a possible graph for trolley **A**:



What could we deduce from this simple graph?
(a) From conservation of momentum and the large drop in speed for **A**, we could show that trolley **B** has a larger mass than trolley **A**.
(b) From the horizontal lines (uniform speed), we can see that there is negligible friction or air resistance acting.

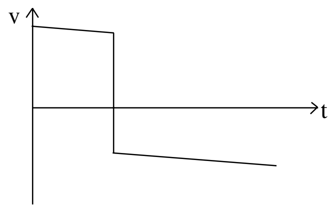
Suppose both trolleys were moving before the collision, and the graph for trolley **A** resembles this:



We could deduce:
(a) **B** was moving towards **A** before the collision.
(b) As the final velocity of **A** is in the opposite direction, **B** was either moving faster than **A**, or **B** has a larger mass, or both.
(c) There is significant friction or air resistance. (The velocity is decreasing as time passes).

> ***Example: Can you use Conservation of Momentum ideas when friction is acting on the trolleys?***
>
> ***Answer:*** *Yes. The change in momentum is almost instantaneous as the collision occurs. Friction should have little effect over this very tiny time interval.*

> ***Exam Hint:*** *When friction is changing the speed of the trolleys before and after a collision, you must use the velocities* ***immediately*** *before and after the collision in any Conservation of Momentum calculations.*

> ***Example: How would you explain this graph for trolley A?***
>
> 
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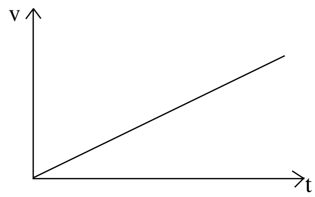
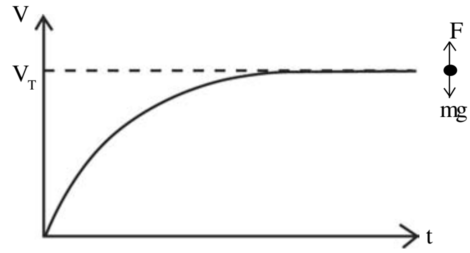
>
> ***Answer:*** *This is similar to the previous collision, except that* ***A*** *accelerates (in the reverse direction) after the collision. The obvious explanation is that it is now travelling downhill (perhaps on a ramp), or with a strong tailwind.*

Objects falling under gravity

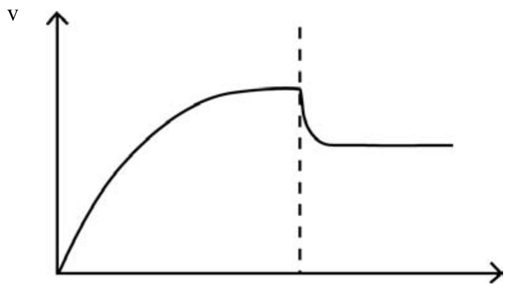
The motion of a skydiver in freefall produces a simple constant acceleration graph at the beginning of her descent.



But as air friction increases with speed:



The terminal velocity $\mathbf{v_T}$ is reached when air resistance increases to balance her weight.

> ***Example: Sketch a v-t graph for a body falling into the sea, after reaching terminal velocity in air.***
>
> ***Answer***
>
> 
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>
> *A second terminal velocity is quickly achieved, when the water resistance and upthrust balance her weight. In which section of the graph are the resistive forces greater than her weight?*

> ***Exam Hint:*** *Be prepared to show the forces acting, and their relative magnitudes, at various points during the object's descent. (A question at the end illustrates this.)*

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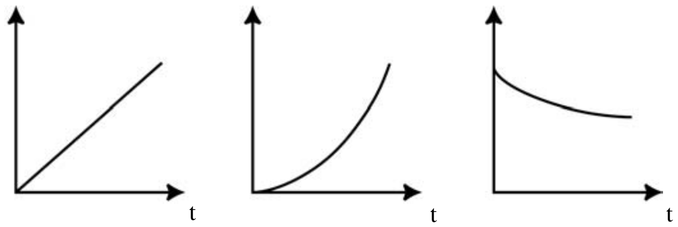
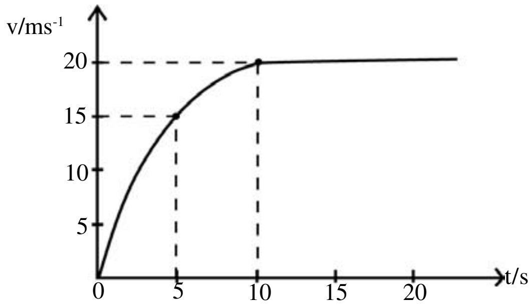
Practice Questions

1. Identify each graph as a velocity-time (v-t) graph or as a displacement-time (d-t) graph:

(a) uniform velocity (b) uniform acceleration (c) projectile with air resistance



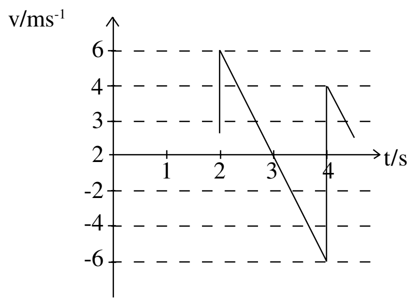
2. This is a graph of a skydiver in the first 20s of her fall.



Find:
(a) the acceleration at the 15s mark
(b) the average acceleration over the first 5 seconds
(c) an estimate of the distance fallen during the 20 seconds
(c) the approximate average speed of descent.

3. For this same skydiver, draw vector diagrams for the forces acting on her after zero, five, and ten seconds.

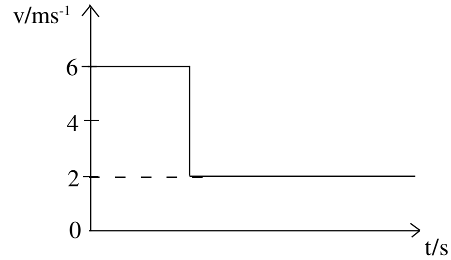
4. This is a section of a v-t graph for a bouncing ball of mass 200g (not necessarily on the Earth).



Find:
(a) the KE of the ball after the first bounce shown
(b) the KE of the ball after the second bounce
(c) the percentage energy lost in this bounce
(d) the height reached after the first bounce
(e) the GPE of the ball at the 3 second mark.

5. (a) Sketch the x-y graph for a projectile fired at an angle of about 30deg above the horizontal. Ignore air resistance.
(b) Sketch the same graph if there was a significant amount of air resistance.
(c) Explain two effects of the air resistance on the path of the projectile.

6. A 10kg trolley collides and sticks to a stationary trolley. This is the graph of the 10kg trolley's motion. Find the mass of the other trolley.



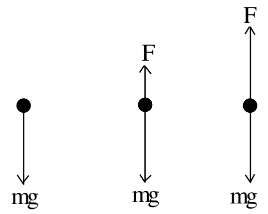
7. For a bouncing ball, sketch a graph of KE against time. Explain your graph.

Answers

1. (a) d-t (b) d-t (c) v-t

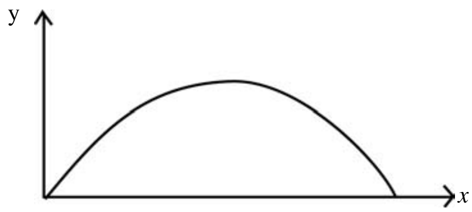
2. (a) zero
(b) $3ms^{-2}$
(c) Approximately 130 + 200 = 330m (adding the areas)
(d) v = d/t = 330 / 20 = $16.5ms^{-1}$

3.

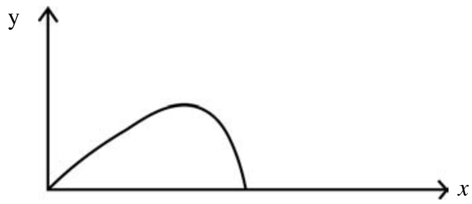


4. (a) KE = $\frac{1}{2} mv^2 = \frac{1}{2} \times 0.2 \times 36 = 3.6J$
(b) 1.6J
(c) % energy lost = $(2.0/36) \times 100 = 56\%$
(d) Height = area = $\frac{1}{2} \times 1.0 \times 6 = 3m$
(e) v = 0, so all KE has become GPE. So GPE = 3.6J

5. (a)



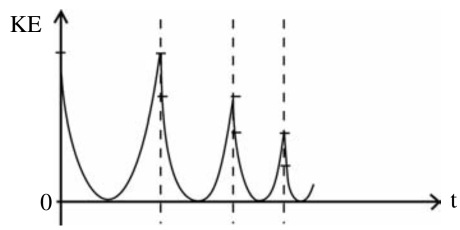
(b)



(c) Slower speed at every point, maximum height less, range less, non-symmetrical path, etc.

6. Conservation of momentum
$10 \times 6 = (10 + m) \times 2 \quad m = 20kg$

7.



Curve as KE $\propto v^2$
KE positive even when v negative, as KE $\propto v^2$
KE lost during each contact with floor.