

Physics Factsheet



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Numerical Calculations & Data Handling

This Factsheet will review basic numerical techniques from GCSE Maths and explain how to carry them out efficiently on a calculator.

1. Percentages

There are three types of percentage calculation you need to know:

- ◆ How to find one number as a percentage of another
- ◆ How to find a percentage of a number
- ◆ How to find percentage error.

I One number as a percentage of another

Expressing something as a percentage is the same as writing it as a fraction with denominator (bottom) 100. Saying $\frac{36}{50} = \frac{72}{100}$ is the same as saying 36 out of 50 is 72%.

This allows us to work out some percentages easily:

Example 1. What is 17 out of 25 as a percentage?

$$\frac{17}{25} = \frac{?}{100}$$

We've multiplied the bottom of the 1st fraction (25) by 4, to get 100
So we must multiply the top of the 1st fraction by 4 as well.

$$\text{So } \frac{17}{25} = \frac{68}{100}$$

So the answer is 68%

Not all percentages involve "easy" numbers, though.

To find y as a percentage of z, you do: $\frac{y}{z} \times 100$

Example 2. What is 0.252 as a percentage of 1.084?

$$\frac{0.252}{1.084} \times 100 = 23.2\%$$

Tip: Don't assume that the larger number will always go on the bottom! Read the question carefully to make sure you have the numbers the right way round.

II Finding a percentage of a number

To do this, you need to

- change the percentage to a fraction by putting it over 100,
- remember that "of" means "multiply"
(so half of 10 means $\frac{1}{2} \times 10$)

Example 3. What is 12.6% of 0.0568?

$$\frac{12.6}{100} \times 0.0568 = 0.0071568$$

III Finding percentage error

You need to learn the formula for this:

$$\% \text{ error} = \frac{\text{experimental value} - \text{actual value}}{\text{actual value}} \times 100$$

Example 4. In an experiment, a student was asked to find the value of g, the acceleration due to gravity. The student obtained the value 8.7ms⁻². Find the percentage error in this value.

$$\% \text{ error} = \frac{8.7 - 9.81}{9.81} \times 100 = -11.3\%$$

Tip. It must be the **real** value on the bottom!

2. Ratios

Ratio calculations involve one of

- ◆ dividing a total in a ratio
- ◆ using the fact that two (or more) quantities are in a certain ratio

I Dividing a total in a ratio

Example 1 illustrates the method used.

Example 1. A sample of an element of mass 5.6g contains isotopes A, B and C in the ratio (by mass) of 2:5:7. Find the mass of isotope C present.

Step 1. Find the total number of "parts" by adding up the numbers in the ratio $2 + 5 + 7 = 14$ parts

Step 2. Find out what one "part" is by dividing the total amount by the number of parts $5.6g \div 14 = 0.4g$ is one part

Step 3. Work out the mass of isotope C by multiplying the mass of one "part" by the number of parts for isotope C in the ratio $0.4g \times 7 = 2.8g$ of isotope C

II Using the fact that quantities are in a certain ratio

This is very similar to dividing a total in a ratio:

Example 2: ³⁷Cl and ³⁵Cl are found in nature in the ratio 1:3. A sample contains 0.891 moles of ³⁵Cl. Find the expected number of moles of ³⁷Cl in this sample.

Step 1. Find out what one "part" is by using the quantity you know. 0.891 moles is 3 parts
So 1 part = $0.891 \div 3 = 0.277$

Step 2. Work out the moles of ³⁷Cl from the number of parts for it in the ratio $1 \times 0.277 = 0.277$ moles

Many percentage and ratio calculations can be done quickly using the “cross method”, which is illustrated below.

Example 1. 2.7% by mass of a sample is isotope A. If there are 0.065g of isotope A in the sample, find the mass of the whole sample.

Step 1. Draw a table, with the headings % and amount. Fill in the figures you know and put a question mark for what you want

%	Amount
2.7	0.065
100	?

The whole sample is 100%

Step 2. Draw a cross

Step 3. Multiply the numbers that are joined and divide by the other number. $100 \times 0.065 \div 2.7 = 2.41\text{g}$

The method for ratios is just the same, but the “%” column is now “ratio”

Example 2. A string is divided in the ratio 5:7. The shorter piece is 62.7 cm long. Find the length of the longer piece.

Ratio Amount

5 62.7
7 ?

So length = $62.7 \times 7 \div 5 = 87.78\text{cm}$.

3. Calculation of Mean and Median

The mean is the normal “average” – where you add all the numbers up and divide by how many there are. The median is the middle number when all the numbers are written in order.

You need to be able to calculate these for:

- ♦ a list of numbers
- ♦ numbers in a frequency table.

Example 1. Find the mean and median of 2, 5, 3, 8, 4, 12

$$\text{Mean} = \frac{2+5+3+8+4+12}{6} = 5\frac{2}{3}$$

For the median, we must arrange them in order: 2, 3, 4, 5, 8, 12
There isn't an actual “middle” number, so we average the two middle numbers:

$$\text{Median} = (4 + 5) \div 2 = 4.5$$

Example 2. The table below summarises the results obtained by a class when measuring the extension of a steel wire under a given load. All measurements were taken to the nearest half millimetre.

Extension (mm)	No. of students obtaining measurement
6.5	3
7.0	5
7.5	8
8.0	7
8.5	4
9.0	1

Find the mean and median of the measurements

Mean

Use the formula:
$$\text{Mean} = \frac{\sum \text{value} \times \text{frequency}}{\sum \text{frequency}}$$

(Σ means “the sum of” – so Σ frequency means “the sum of the frequencies”)

Here, the values are the measurements, and the frequencies (= numbers of people or items) are the numbers of students getting each measurement.

$$\text{So mean} = \frac{6.5 \times 3 + 7.0 \times 5 + 7.5 \times 8 + 8.0 \times 7 + 8.5 \times 4 + 9.0 \times 1}{3 + 5 + 8 + 7 + 4 + 1} = 7.625$$

Median

Step 1. Work out a running total of the frequencies (called “cumulative frequency”)

Extn	No. of students	Cumulative Frequency
6.5	3	3
7.0	5	3 + 5 = 8
7.5	8	3 + 5 + 8 = 16
8.0	7	3 + 5 + 8 + 7 = 23
8.5	4	3 + 5 + 8 + 7 + 4 = 27
9.0	1	3 + 5 + 8 + 7 + 4 + 1 = 28

Step 2. Find out where the “halfway point” is – do this by adding 1 to the total of the frequencies and dividing by 2:

$$\text{Halfway point is } \frac{28+1}{2} = 14.5, \Rightarrow \text{between } 14^{\text{th}} \text{ \& } 15^{\text{th}} \text{ values}$$

Step 3. Go down the table until you get to the first cumulative frequency equal to or greater than the halfway point. Read across to the value. This is the median.

The first cumulative frequency above 14.5 is 16.
This goes with the extension 7.5mm – so this is the median.

Sometimes you may be given a **grouped frequency table** – instead of a single number on the left, you have a range of numbers, like 15.5 – 19.5. Finding the mean for these is just like finding it for the simple frequency tables – only the values you use are the **midpoints**. You can find the median class using the method given above; there are other methods for finding an estimate of the median value, but you will not be required to use them.

Example 3. Find the mean of the data given below.

Value	Frequency
$6.5 \leq x < 7.5$	5
$7.5 \leq x < 8.5$	8
$8.5 \leq x < 10.5$	4
$10.5 \leq x < 15.5$	3

First we find the midpoints of each class (by adding its endpoints and dividing by 2)

Value	Midpoint	Frequency
$6.5 \leq x < 7.5$	$(6.5 + 7.5) \div 2 = 7.0$	5
$7.5 \leq x < 8.5$	$(7.5 + 8.5) \div 2 = 8.0$	8
$8.5 \leq x < 10.5$	$(8.5 + 10.5) \div 2 = 9.5$	4
$10.5 \leq x < 15.5$	$(10.5 + 15.5) \div 2 = 13.0$	3

Now we use the midpoints to find the mean in exactly the same way as example 2:

$$\text{mean} = \frac{7.0 \times 5 + 8.0 \times 8 + 9.5 \times 4 + 13.0 \times 3}{5 + 8 + 4 + 3} = 8.8$$

4. Calculator Use

Many students who understand how to do a calculation end up with the wrong answer through not using their calculator correctly! This section will highlight some common errors, explain how to avoid them and give some hints on calculator use.

One of the most important things to remember is that you should always check your answer is the right sort of size – do this by making a rough estimate before you start by rounding all the numbers. For example, if you are calculating 0.0652×24.3 , round it to 0.07×20 , which is 1.4. You obviously won't get exactly 1.4 as your answer, but you would know that if you got 250, it was probably wrong!

I Percentages and Ratios

Doing percentage calculations on a calculator shouldn't cause problems in general, (although watch out for fractions problems – see below) but it is generally safer to **avoid** using the % key unless you are 100% confident you know exactly what you are doing with it.

Ratios are also fairly fool-proof except for division/fraction problems (below).

II Calculating the mean

Scientific and graphical calculators will have a function built in to them to allow you to calculate the mean of a list of numbers or of a frequency distribution. This Factsheet cannot give you a definitive guide to doing this on your calculator – all calculators are different! – but will give some ideas on where to start and things to watch out for.

Scientific Calculators

- ◆ You usually have to go into SD mode.
- ◆ Before you start, make sure the statistical memory is clear – the button for this is often called “SAC” and can usually be accessed by pressing the 2^{nd} /shift button and AC
- ◆ To enter a list of numbers, type in each number, then press “DATA”. This is found in different places on different calculators – but near the M+ button may be a good place to look.
- ◆ If you want to put in numbers and frequencies, on most calculators you type in number \times frequency then “DATA”. (NB: you must put in 2×4 , say, not 8!)
- ◆ When you've typed in all your numbers, it's a good idea to check how many numbers you've put in, to make sure you haven't missed any out. To do this, look for the “n” above one of the number buttons – press the 2^{nd} /shift button to get to it. (Some calculators tell you this as you go along)
- ◆ To find the mean, look for the “ \bar{x} ” above one of the number buttons. Press the 2^{nd} /shift button to get to it.
- ◆ Check that your answer is sensible!
- ◆ Clear the data when you have finished.

Graphical Calculators

- ◆ Most graphical calculators have a STAT mode for statistical calculations. This allows you either to enter the data in a table, or prompts you for each value and its frequency.
- ◆ You need to go into the SETUP option within the STAT mode to change between putting in a list of values or values and frequencies.
- ◆ When you've finished putting in the data, you need to use the menu to get all the calculations. Unfortunately, this is different on all the different makes! But almost all give you a big list of figures – the mean, the sum of the values, the sum of the squares of the values... It may be stating the obvious, but make sure you write the correct one down!
- ◆ If the value you've got doesn't make sense, then probably you either have some earlier data in there, or the calculator thinks it's working with a list of values when you want it to use frequencies.

III General Calculations and Fractions

Standard scientific calculators almost always perform calculations using BIDMAS.

This means that calculations are done in the order:

Brackets	Indices	Division	Addition
		Multiplication	Subtraction

So if you were to type in $2 + 5 \times 3$, it will do 5×3 first, then add 2, to get the answer 17. If you type in $(2 + 5) \times 3$, it will give the answer 21.

If your calculation involves division and multiplication, or addition and subtraction, the calculator does them in the order they are put in (so it works from left to right)

For example, $12 - 5 + 3$ gives 10, and $24 \div 3 \div 2$ gives 4.

Other functions, like e^x , $\log x$, $\ln x$, $\sin x$ etcetera, come at the “indices” level – so putting in $\sin 30 \times 2$ will give the sine of 30° , multiplied by 2 (which gives 1), but $\sin(30 \times 2)$ will give the sine of 60° (0.866...)

Most of the time, this means that provided you have written your original calculation correctly, using brackets where necessary, you will get the right answer. However, the major exception to this rule is when a calculation is written in fraction form.

The reason calculations written as fractions are a problem is that fractions allow you to miss off brackets without noticing.

For example, $\frac{2.659 + 30.052}{8.3}$ actually means $(2.659 + 30.052) \div 8.3$

If you enter $2.659 + 30.052 \div 8.3$ into your calculator, it does $30.052 \div 8.3$ first, then adds on 2.659 – so you don't get the right answer!

Similarly, $\frac{0.085}{2.63 \times 3.6}$ really means $0.085 \div (2.63 \times 3.6)$.

If you enter $0.085 \div 2.63 \times 3.6$ into your calculator, it does $0.085 \div 2.63$, then multiplies the answer by 3.6.

Avoiding the problem is very easy – **whenever you have a calculation written as a fraction, put all of the top in brackets, then put in a division sign, then all of the bottom in brackets.**

IV The Fraction Key

You can use the fraction key to get answers in terms of fractions, but beware:

- ◆ If you are using it to work out a calculation written as a fraction (see above), you **still** need the **brackets**.
- ◆ If you want something like $(\frac{3}{4})^3$, you **must put in the brackets** otherwise it will only cube the bottom of the fraction!

NB: Some graphical calculators do not work like this.

V Using the memory

If you have a lengthy calculation to perform, it may be complicated to type it all in at once, putting brackets in the right places. Often it's easier to work out one part first, then use the answer in the rest of the calculation.

This works well if the answer is an “easy” number, like 26, or 0.03 or similar, since you can type the exact answer back into the calculator. But if the answer is something like 0.67843201..., then you may be tempted to round it before using it in the next step of the calculation, or you may make a mistake writing it down or re-typing it in. In this case, you are much better off using the memory in your calculator.

In scientific calculators, the memory is accessed by using the M+ (or similar) key. You must make sure you clear the memory (MC) before putting a number in, or it will just add your number to whatever is in the memory! When you want to use the contents of the memory, you type the calculation in the normal way, but press the recall memory (MR) key whenever you want the number in the memory.

Most graphical calculators have several memories, which are accessed by using a key like "STO" or "→", then a letter. You can recall the contents of each letter memory by pressing the letter key, and use it in calculations in the same way as for scientific calculators. The exact keys you use vary from calculator to calculator – but it is never difficult to use, and it is worthwhile – so spend some time with your calculator manual!

Questions

- What is 3 as a percentage of 4.02?
 - What is 0.078 as a percentage of 0.892?
 - What is 6.752 as a percentage of 5.063?
 - A student obtained 64 marks out of 68. What is this mark as a percentage?
- What is 20.5% of 32?
 - What is 117% of 0.062?
 - A student obtained 17.5% in a test out of 240. How many marks did the student obtain?
 - 45 readings were taken in an experiment. 20% of them were discarded due to errors. How many readings was this?
- A student measuring g obtained the value 9.76ms^{-2} . Calculate the percentage error made by this student, using $g = 9.81\text{ms}^{-2}$
 - A second student made a percentage error of 12% when measuring g . Given that this student's value was too high, calculate the value s/he obtained.
 - In an experiment to measure the density of an unknown liquid, the result obtained was 0.03gcm^{-3} too high. Given that this was an error of 2%, find the true density of the liquid.
 - A student measures the length of an object as 9cm (to the nearest cm).
Assuming this value is correct to the given accuracy,
 - What is the smallest possible value of the true length of the object?
 - What is the maximum percentage error that the student could be making?
- A sample of compound X is known to contain elements A and B in the ratio (by mass) of 46:71.

 - There are 0.674g of the compound. Calculate the mass of each element present.
 - In another sample of the same compound, there are 1.83g of element A. Find the mass of element B in this sample.

Compound Y contains elements A, B and C in the ratio 46:71:32. It is known to react to produce compound X and element C.

 - Calculate the mass of compound X produced from 3.8g of compound Y.
 - Calculate the mass of compound Y required to produce 0.92g of compound X.

- Find the mean and median of the following:
3, 8, 2, 4, 5, 6, 1, -3, 2, -10

- Find the mean and median of the following:

Value	Frequency
21	43
32	16
49	28
51	14
58	2

- Find the mean of the following:

Value	Frequency
$19.5 \leq x < 29.5$	12
$29.5 \leq x < 34.5$	13
$34.5 \leq x < 39.5$	23
$39.5 \leq x < 49.5$	11
$49.5 \leq x < 69.5$	4

- Use your calculator to find the answer to the following calculations:

$$\text{a) } \frac{3.8 + 2.09}{62.8}$$

$$\text{b) } \frac{3.2}{4.09 - 2.15}$$

$$\text{c) } \frac{3.609 \times 0.08}{\pi \times 3.24}$$

$$\text{d) } \frac{2.6 + \sqrt{2.6^2 - 4 \times -2 \times 0.1}}{2 \times 0.1}$$

Hint: Work out the square root first & put the answer in memory for d).

Answers

- 74.6%
 - 8.74%
 - 133%
 - 94.1%
- 6.56
 - 0.07254
 - 42
 - 9
- 0.510%
 - 11.0ms^{-2}
 - 1.5gcm^{-3}
 - i) 8.5cm ii) 5.88%
- 0.265g of X, 0.409g of Y
 - 2.82g c) 2.98g d) 1.17g
- mean 1.8 median 2.5 b) mean 36.2 median 32 c) 36.3
- 0.0938 b) 1.65 c) 0.0284 d) 39.7

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